Signal transmission in one-way coupled bistable systems: Noise effect

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Signal transmission in a nonlinear medium described by one-way coupled bistable systems subject to uncorrelated noise is investigated. It is found that noise can play an active role to effectively improve the signal transmission. Stochastic resonance behavior can appear in the signal transmission process, and at the optimal noise intensity one can find the largest output signal amplitude and the largest signal transmission distance. In particular, in certain cases undamped signal transmission can be stimulated by the combined action of both injected signal and noise, while the signal damps exponentially with space distance without noise.
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Recently, the investigation of extended systems has become a hot topic in the field of nonlinear science [1–9]. The extremely rich behavior of dynamics, patterns, and the significant responses of these systems to external excitations have attracted much attention. The following coupled systems have been most extensively analyzed in this regard:

$$\dot{x}_i = f(x_i) - \frac{\mu+\gamma}{2}(x_i - x_{i-1}) - \frac{\mu-\gamma}{2}(x_i - x_{i+1}), \qquad i = 1,2,\ldots,N, \tag{1}$$

where $f(x_i)$ includes local nonlinear dynamics, and the couplings are simply linear with μ representing diffusive coupling and γ gradient coupling; both are important and popular in practice. By setting $\mu=\gamma$ we obtain one-way coupled systems, which have attracted great interest and have been extensively investigated in recent years for describing open flow systems [10–12] and one-directional driving dynamical chains [13,14] with preferential coupling direction down string. In the present paper, we focus on this one-way coupling case.

With the one-way coupled systems, an important problem is how the down string sites of the system respond to an excitation applied at an upper string site. In particular, we inject a signal to the first site of the system, it is interesting to ask how this signal can transmit in the coupled medium. Moreover, the medium may be subject to noise, then one may further ask how noise can influence the signal transmission process. Usually, one expects that noise may spoil signal and make signal transmission difficult. However, in the recent two decades, we have learned a lot about the active role played by noise in enhancing coherence, so-called stochastic resonance (SR) [15–27] (for the latest review see Ref. [25]). It is then significant to ask whether we can also find the SR-like behavior in the signal transmission problem, and how this behavior influences the signal propagation if we can. In the following we find the influence of noise on the signal transmission can be rather positive.

Let us fix $\mu=\gamma$, $x_{i=0}=0$, and take $f(x)=ax-x^3$ in Eqs. (1) and specify our model as

$$\dot{x}_1 = (a-\mu)x_1 - x_1^3 + \Gamma_1(t), \tag{2a}$$

$$\dot{x}_i = (a-\mu)x_i - x_i^3 + \mu x_{i-1} + \Gamma_i(t), \qquad i = 2,3,4,\ldots,N. \tag{2b}$$

Moreover, we fix $a-\mu=1$ and inject the first cell by a sinusoidal force, then Eqs. (2) can be modified to

$$\dot{x}_1 = x_1 - x_1^3 + \Gamma_1(t) + A\cos\Omega t, \tag{3a}$$

$$\dot{x}_i = x_i - x_i^3 + \mu x_{i-1} + \Gamma_i(t), \qquad i = 2,3,4,\ldots,N, \tag{3b}$$

where $\Gamma_i(t)$ are white and spatially uncorrelated noises $\langle \Gamma_i(t)\rangle = 0$, $\langle \Gamma_i(t)\Gamma_j(t')\rangle = 2D\delta_{ij}\delta(t-t')$. Throughout this paper we fix $\Omega=0.05$, then we have A, μ, and D as our three control parameters. Equation (3a) has been extensively used for studying the SR problem. Now we focus on how the signal $A\cos\Omega t$ affects, through the couplings (or say, one-way drivings), the outputs of the sites down string. The quantity chosen for describing the signal transmission in the bistable medium is B_i:

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$$B_i = C_i(\Omega) - [C_i(\Omega+\Delta\Omega) + C_i(\Omega-\Delta\Omega)]/2,$$

$$C_i(\omega) = \sqrt{\left\{(1/T)\int_{T_0}^{T_0+T}[x_i(t)-\overline{x_i}]\cos(\omega t)dt\right\}^2 + \left\{(1/T)\int_{T_0}^{T_0+T}[x_i(t)-\overline{x_i}]\sin(\omega t)dt\right\}^2}, \tag{4}$$

$$\overline{x_i} = \frac{1}{T}\int_{T_0}^{T_0+T} x_i(t)dt,$$

where T_0 is long enough to forget the transient, and B_i is the signal included in the x_i output, while $C_i(\Omega+\Delta\Omega)$ and $C_i(\Omega-\Delta\Omega)$ are the noise background in the vicinity of Ω. In this paper we fix $\Delta\Omega = 0.1\Omega$ (the particular value $\Delta\Omega$ does not affect all the following results if $2\pi T \ll \Delta\Omega \ll \Omega$). For simulation, we take long enough T ($T = 1002\pi/\Omega$, further increasing T does not affect the results) to guarantee that B_i is truly the output signal. It is obvious that B_i cannot be obtained analytically for finite noise intensity. We have to focus on numerical simulations to detect the noise effect. The essential significance of the results in this paper is that the transmission efficiency is unexpectedly sensitive to the noise influence.

First we consider the signal transmission without noise, $D=0$. In Figs. 1(a) and 1(b) we plot B_i versus i for various A and μ. In all cases considered, the signal can only influence few sites with very small i. B_i is invisible in the figures for $i \geqslant 10$ and is invisible even in the computer precision for $i \geqslant 20$ at $\mu = 0.5$ and $i \geqslant 30$ at $\mu = 4.0$. Therefore, the signal transmission is local, and it actually damps exponentially

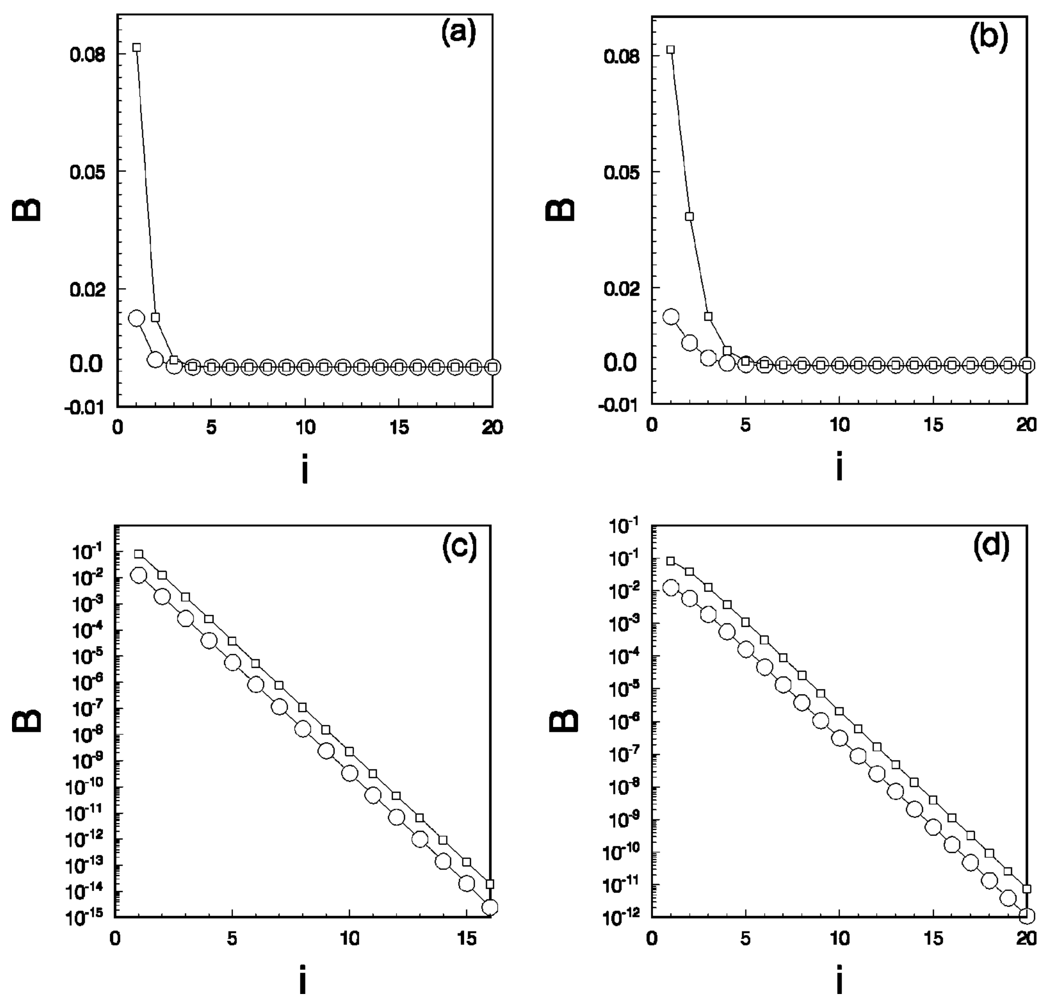

FIG. 1. $D=0$. (a),(b) The output signal amplitudes B_i vs i. (a) $\mu=0.5$, the squares and circles represent $A=0.3$ and 0.05, respectively. (b) The same as (a) with $\mu=4$. (c),(d) The same as (a) and (b), respectively, with B plotted logarithmically. A clear exponential decay law of Eq. (5) is justified. The slope in (c) is about $\alpha \approx 1.95$ while it is about $\alpha \approx 1.25$ in (d).

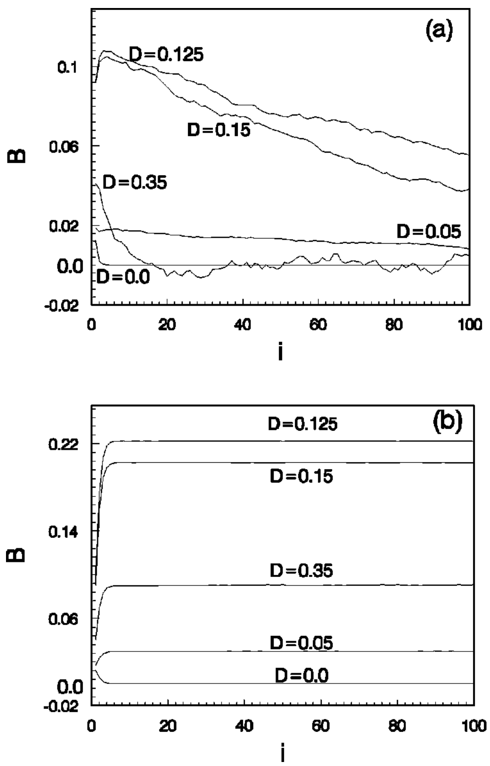

FIG. 2. $A=0.05$. Data (in Figs. 2–5) are obtained by averaging the results of 50 runs. (a),(b) The same as Figs. 1(a) and 1(b), respectively, with various nonzero noise. It is obvious that proper noise can enhance signal transmission efficiency, and there exists an optimal noise for each μ yielding the largest output signal amplitude. In (b) undamped signal transmission is observed.

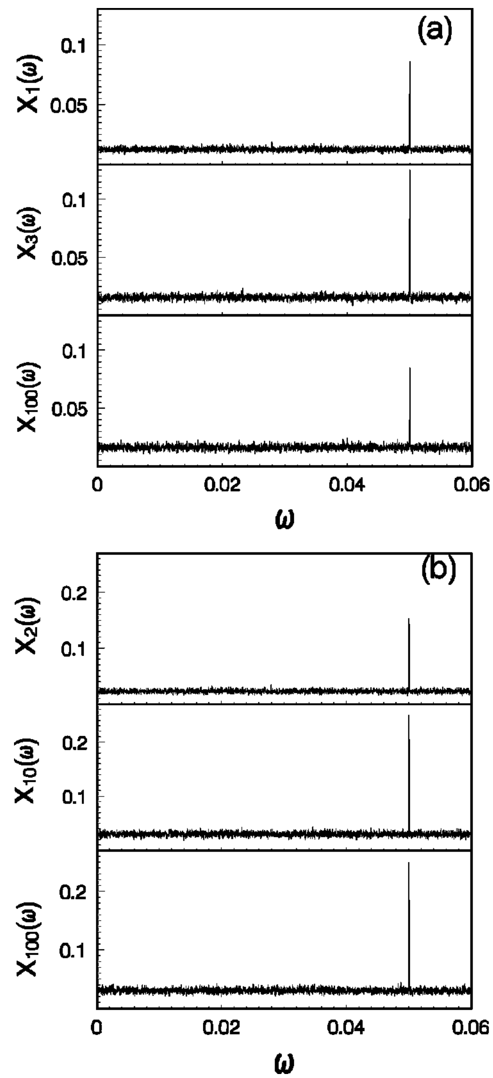

FIG. 3. (a) The spectra of $x_i(t)$ for various i. $A=0.05$, $\mu=0.5$, $D=0.125$. (b) The same as (a) with $\mu=4$. In (b) the amplitude and quality of signals $x_i(t)$ are practically unchanged with i as i is sufficiently large.

with respect to space distance from the first site. An experiential formula for the damp reads

$$B_i \approx C(A)\exp[-\alpha(\mu)i], \tag{5}$$

where $\alpha \approx 1.95$ and 1.25 for Figs. 1(a) and 1(b), respectively, and C depends on the parameter A [see Figs. 1(c) and 1(d)].

Now we study the influence of noise on the signal transmission process. All data in Figs. 2–5 are given by averaging the results of 50 runs. In Figs. 2(a) and 2(b) we fix $A=0.05$, and take $\mu=0.5$ and 4.0, respectively, and do the same as Figs. 1(a) and 1(b) with different nonzero D. Several interesting features are observed. (i) In Fig. 2(a), for not too large noise (i.e., except $D=0.35$) the decay of B_i with i becomes roughly linear rather than exponential in Fig. 1, and then the signal can be transmitted much farther in the bistable medium than that without noise. (ii) There exists an optimal noise for the most effective transmission; both too low and high noises may decrease the signal transmission distance or reduce the output signal amplitude B_i. (iii) Above certain μ, suitable noise may produce undamped signal transmission, i.e., the signal may propagate in the bistable medium for an infinitely long distance (in our numerical simulation, we have tested $N=200$, and no trace of B_i damping could be found at the ending site). In Figs. 3(a) and 3(b) we plot the spectra of several sites at $A=0.05$ and $D=0.125$ for different μ, and undamped signal transmission is found in Fig. 3(b). Note that for $A=0.05$ we can find a very large transmitted signal at $i=100$ in Fig. 3(b); this is in sharp contrast with Fig. 1(b) where for much larger A ($A=0.3$) the trace of the signal is completely lost at $i=10$ without noise.

Figures 4 and 5 are used to manifest the above features (ii) and (iii) more clearly. In Figs. 4(a) and 4(b) we plot B_i versus D for various i by fixing $A=0.05$ and taking $\mu=0.5$ and 4, respectively. A clear stochastic resonance behavior is justified in each curve. For small μ (e.g., $\mu=0.5$) the SR curve has the highest peak at a certain finite i (e.g., $i=3$ in our case), for both smaller and larger i, the SR peaks become lower. For sufficiently large μ (e.g., $\mu=4$), the SR peak increases as i increases, and it saturates to a certain value for large i (the data for $i=50$ and $i=100$ are identical) and maintains an undamped height for any large i tested, i.e., noise induced undamped signal transmission is further identified. For testing the behavior of the signal propagation in the bistable medium, we present Fig. 5. In Fig. 5(a) we plot M versus D at $A=0.05$ for different μ, where M represents the signal transmission distance, defined by the total number of sites satisfying both conditions

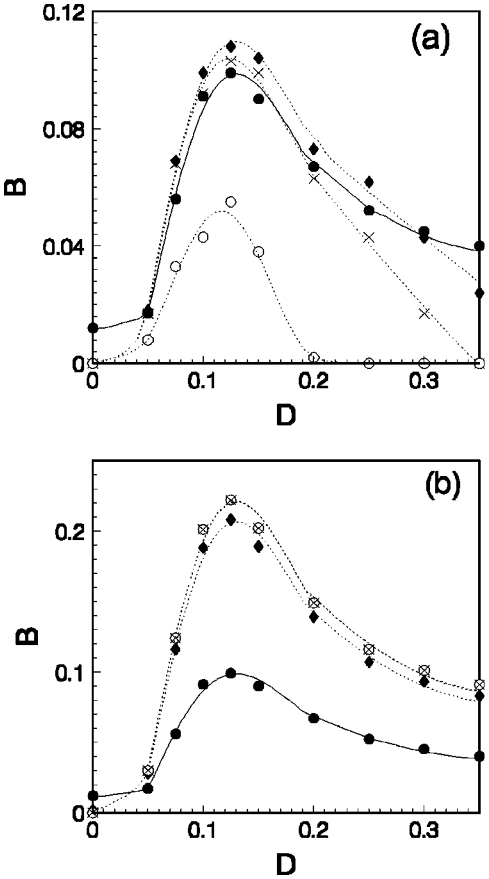

FIG. 4. $A = 0.05$. (a) B_i vs D for different i at $\mu = 0.5$. The disks, diamonds, crosses, and circles represent B_1, B_3, B_{10}, and B_{100}, respectively. (b) The same as (a) with $\mu = 4$. In both (a) and (b) stochastic resonance is clearly seen, and in (b) the SR curves are practically identical for different i as i is large; that further verify the undamped signal transmission.

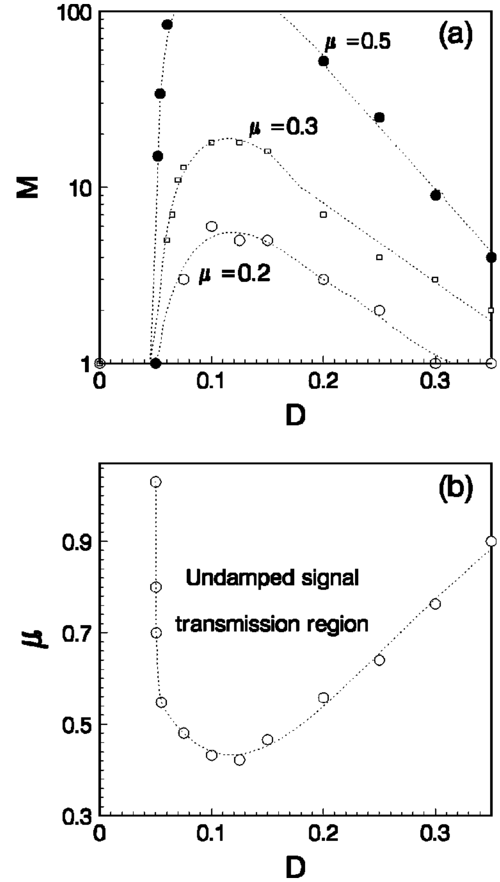

FIG. 5. (a) M vs D for different μ, where M is the number of sites receiving the transmitted signal, satisfying Eqs. (6). (b) The region of undamped signal transmission (defined by $M > 100$) in the μ-D plane.

$$B_i > 0.02 \tag{6a}$$

and

$$\frac{2C_i(\Omega)}{C_i(\Omega+\Delta\Omega)+C_i(\Omega-\Delta\Omega)} > 1.5, \tag{6b}$$

where the condition (6b) is needed for detecting signal when noise is very large. [With noise, fluctuation of B_i is inevitable due to finite integration time T. At very large D this fluctuation can be large, which makes the first condition (6a) doubtful.] The two conditions Eqs. (6a) and (6b) together can identify the signal in the output of any site without any ambiguity. In Fig. 5(a) for small μ we again find that nice stochastic resonance curves, i.e., the signal can transmit farthest for certain optimal noise. For large μ, M can reach undamped signal propagation in a certain segment of D. In Fig. 5(b) we plot the region for undamped signal transmission in the D-μ plane at $A = 0.05$ (in our simulation we identify undamped signal transmission by $M > 100$). We find that undamped signal transmission does not exist for small μ and small D. As μ increases from zero over a critical value, the undamped signal transmission appears first at the optimal D (the noise intensity for stochastic resonance, now it is $D \approx 0.125$ in our case), and then the D range for undamped signal transmission can be enlarged by further increasing μ.

In conclusion we have investigated the signal transmission process in a medium represented by one-way coupled bistable systems. The influence of noise on the transmission behavior has been analyzed numerically in detail. It is shown that properly applying noise can considerably improve the signal transmission efficiency. Stochastic resonance behavior can be clearly seen in the signal transmission process, and at certain optimal noise intensity the largest output signal amplitude (Figs. 2, 3, and 4) and the largest signal transmission distance [Fig. 5(a)] appear. In particular, we find that for relatively strong coupling, noise can induce undamped signal transmission, while without noise the signal damps exponentially down string and the signal transmission distance is very small.

However, some open problems still exist, which are worth investigating further. First, it is very interesting to examine what will happen if the signal carries certain information. We expect that the approach of the present paper may work also in the case of an informative signal. Second, the effect of noise-enhancing signal transmission efficiency is believed to be general for bistable systems, regardless of the particular form of the inner dynamics of the sites. It is also interesting to see whether the bistability is a necessary condition for the noise-enhanced signal transmission efficiency. Moreover, in this paper we use one-way coupled systems. However, the one-way coupling is not the necessary condition. The similar phenomenon can also appear in bidirectionally coupled systems. Nevertheless, as the back-directional coupling increases, the effect of noise-enhancing signal decreases. The enhancement is strongest for one-way coupling [$r=\epsilon$ in Eq. (1)] and is weakest for symmetric coupling [$r=0$ in Eq. (1)]. Finally, it is of significance to apply the ideas in this paper to practical systems. We believe that optical systems and neural network systems are the best candidates to do so since both bistability and noise are common there and signal transmission properties are an important issue for the investigation of these systems.

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